OCR
Oxford Cambridge and RSA

Thursday 26 May 2022 – Afternoon

AS Level Further Mathematics A

Y534/01 Discrete Mathematics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.
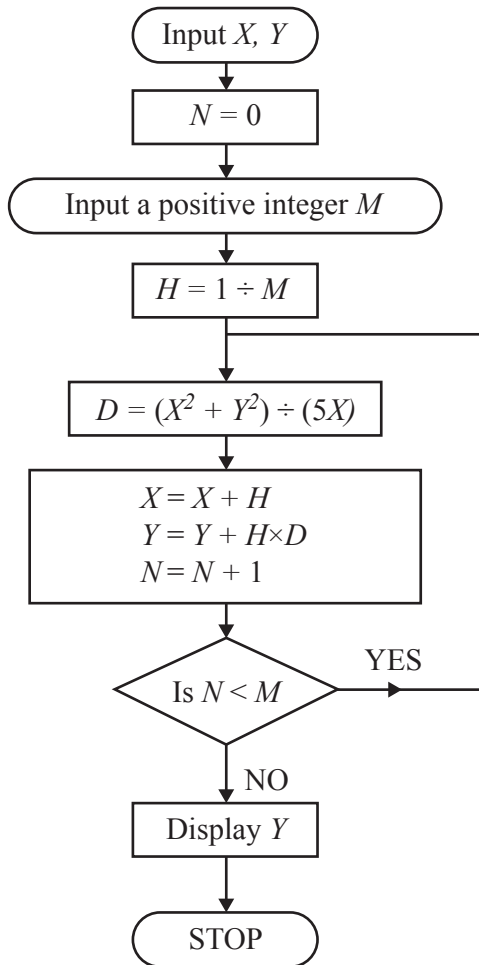
INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

## ADVICE
- Read each question carefully before you start your answer.

1   The flowchart below has positive inputs *X, Y* and *M*.

```
              ┌──────────────┐
              (  Input X, Y  )
              └──────┬───────┘
                     ▼
              ┌──────────────┐
              │    N = 0     │
              └──────┬───────┘
                     ▼
        ┌───────────────────────────────┐
        ( Input a positive integer M     )
        └──────────────┬────────────────┘
                       ▼
              ┌──────────────┐
              │  H = 1 ÷ M   │
              └──────┬───────┘
                     ▼                    ┌──────┐
        ┌──────────────────────────┐      │      │
        │ D = (X² + Y²) ÷ (5X)      │      │      │
        └──────────────┬───────────┘      │      │
                       ▼                   │      │
        ┌──────────────────────────┐      │      │
        │  X = X + H                │      │      │
        │  Y = Y + H×D              │      │      │
        │  N = N + 1                │      │      │
        └──────────────┬───────────┘      │      │
                       ▼          YES      │      │
                   ◇─────────◇────────────►┘      
                   ◇ Is N < M ◇
                   ◇─────────◇
                       │ NO
                       ▼
              ┌──────────────┐
              │  Display Y   │
              └──────┬───────┘
                     ▼
              ┌──────────────┐
              (    STOP       )
              └──────────────┘
```

Input X, Y

$N = 0$

Input a positive integer M

$H = 1 \div M$

$D = (X^2 + Y^2) \div (5X)$

$X = X + H$
$Y = Y + H \times D$
$N = N + 1$

Is $N < M$ YES

NO

Display Y

STOP

(a) Trace through the flowchart above using the inputs $X = 1$, $Y = 2$ and $M = 2$. You only need to record values when they change. **[3]**

(b) Explain why the process in the flowchart is finite. **[1]**

2 The activities involved in a project and their durations, in hours, are represented in the activity network below.
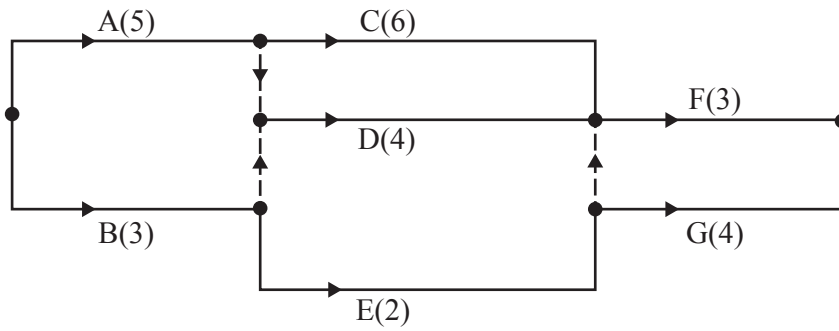
(a) Carry out a forward pass and a backward pass through the network. **[3]**

(b) Calculate the float for each activity. **[2]**

A delay means that activity B cannot finish until t hours have elapsed from the start of the project.

(c) Determine the maximum value of t for which the project can be completed in 16 hours. **[2]**

3 **(a)** The list below is to be sorted into **increasing** order using bubble sort.

52   38   15   61   27   49   10   33   96   74

  (i) Determine the list that results at the end of the first, second and third passes.
  You do **not** need to show the individual swaps in each pass. **[2]**

  (ii) Write down the number of comparisons and the number of swaps used in each of these passes. **[2]**

(b) The list below is to be sorted into **increasing** order using shuttle sort.

52   38   15   61   27   49   10   33   96   74

  (i) Determine the list that results at the end of the first, second and third passes.
  You do **not** need to show the individual swaps in each pass. **[2]**

  (ii) Write down the number of comparisons and the number of swaps used in each of these passes. **[2]**

(c) Use the results from parts **(a)** and **(b)** to compare the efficiency of bubble sort with the efficiency of shuttle sort for the first three passes of this list.
You do **not** need to consider what happens after these three passes. **[2]**

4 Kareem and Sam play a game in which each holds a hand of three cards.

- Kareem's cards are numbered 1, 2 and 5.
- Sam's cards are numbered 3, 4 and 6.

In each round Kareem and Sam simultaneously choose a card from their hand, they show their chosen card to the other player and then return the card to their own hand.

- If the sum of the numbers on the cards shown is even then the number of points that Kareem scores is $2k$, where k is the number on Kareem's card.
- If the sum of the numbers on the cards shown is odd then the number of points that Kareem scores is $4 - s$, where s is the number on Sam's card.

(a) Complete the pay-off matrix for this game, to show the points scored by Kareem. **[3]**

(b) Write down which card Kareem should play to maximise the number of points that he scores for each of Sam's choices. **[2]**

(c) Determine the play-safe strategy for Kareem. **[2]**

(d) Explain why Kareem should never play the card numbered 1. **[1]**

Sam chooses a card at random, so each of Sam's three cards is equally likely.

(e) Calculate Kareem's expected score for each of his remaining choices. **[2]**

5 A baker makes three types of jam-and-custard doughnuts.

- Each batch of type X uses 6 units of jam and 4 units of custard.
- Each batch of type Y uses 7 units of jam and 3 units of custard.
- Each batch of type Z uses 8 units of jam and 2 units of custard.

The baker has 360 units of jam and 180 units of custard available.

The baker has plenty of doughnut batter, so this does not restrict the number of batches made.

From past experience the baker knows that they must make at most 30 batches of type X and at least twice as many batches of type Y as batches of type Z.

Let x = number of batches of type X made
y = number of batches of type Y made
z = number of batches of type Z made.

(a) Set up an LP formulation for the problem of maximising the total number of batches of doughnuts made. **[5]**

The baker finds that type Z doughnuts are not popular and decides to make zero batches of type Z.

(b) Use a graphical method to find how many batches of each type the baker should make to maximise the total number of batches of doughnuts made. **[4]**

(c) Give a reason why this solution may **not** be practical. **[1]**

The baker finds that some of the jam has been used so there are only k units of jam (where $k < 360$).
There are still 180 units of custard available and the baker still makes zero batches of type Z.

(d) Find the values of k if **exactly one** of the other (non-trivial) constraints is redundant. Express your answer using inequalities. **[3]**

6 Eight footpaths connect six villages. The lengths of these footpaths, in km, are given in the table.

Villages connected	A B	A D	B E	B F	C D	C E	D E	E F
Length of footpath, km	3	2	4	6	5	7	3	1

(a) The shortest route from B to C using these footpaths has length 10 km.
Without using an algorithm, write down this shortest route from B to C. **[1]**

(b) Use an appropriate algorithm to find the shortest route from A to F. **[3]**

(c) Write down all the pairs of villages for which the shortest route between them uses at least one footpath that is **not** in the minimum spanning tree for the six villages. **[5]**

7 **(a)** List the 15 partitions of the set {A, B, C, D, E} in which A and E are in the same subset. **[3]**

(b) By considering the number of subsets in each of the partitions in part **(a)**, or otherwise, explain why there are 8 partitions of the set {A, B, C, D, E} into two subsets with A and E in different subsets. **[2]**

Ali says that each of the 15 partitions from part **(a)** can be used to give two partitions in which A and E are in different subsets by moving E into a subset on its own or by moving E into another subset.

(c) **(i)** By considering the partition from part **(a)** into just one subset, show that Ali is wrong. **[1]**

(ii) By considering a partition from part **(a)** into more than two subsets, show that Ali is wrong. **[1]**

END OF QUESTION PAPER